M. Phil. Examination
April / May - 2003
Statistics (Compulsory) : Paper-II
(Theory of Distributions)

Time : 3 Hours] [Total Marks : 100

Instructions : (i) Answer the following questions.
(ii) Each question carries 20 marks.
(iii) Scientific calculators and statistical tables can be used.

1 (a) What is intervened distribution? Give some practical situations where intervened distributions may arise.

(b) Derive pmf of intervened Poisson distribution. Obtain its mean and variance.

OR

1 (a) Define a spherical family of p-variate distributions. Show that p-variate normal distribution is a member of this family.

(b) Define Elliptical family of distributions.
Let \( x \) ~ \( E_p(\mu, \Sigma) \). Obtain

(i) \( E(x) \) and \( V(x) \)

(ii) \( \phi_{\Sigma}(t) \) \( \forall \) \( t \in \mathbb{R}^p \).

(c) Define Wishart distribution. Write its important properties. State and prove additive property of this distribution.
2 (a) Define intervened geometric distribution. Point out the real life situations where this distribution is recommended.

(b) For an IGD show that the \( k \)th factorial moment \( \mu_{[k]}' \) is given by

\[
\mu_{[k]}' = \frac{(1-q)(1-\rho q)}{q(1-\rho)} \left[ \frac{q^k}{(1-q)^{k+1}} - \frac{(\rho q)^k}{(1-\rho q)^{k+1}} \right]
\]

for \( k = 1, 2, 3, \ldots \) and \( q \) and \( \rho \) are the parameters of the distribution.

(c) Discuss the method of maximum likelihood to estimate the parameter of the IGD.

OR

2 (a) Define a generalised gamma distribution. Explain how it reduces to classical exponential and gamma distributions. How would you estimate the parameters of Generalised Gamma Distribution?

(b) Define Generalised Possion Distribution (GPD). How will you obtain GPD as a particular case of modified power series distribution. Obtain the mean and variance of GPD.

3 (a) Define Lagrangian Beta and Gamma distributions.

(b) State and prove any two properties of Lagrangian gamma distribution.

(c) Define negative multinomial distribution. Obtain its moment generating function.

OR

3 (a) Define a mixture distribution. Prove N-S condition for the identifiability of a class of mixture distributions.
(b) State the different methods for estimating the parameters of a mixture distribution and point out their merits and demerits.

(c) Using Rider’s method obtain the estimate of the parameters of a mixture distribution with density.

\[ f(x) = p_1 \mu_1^{-1} \exp\left(-\frac{x}{\mu_1}\right) + p_2 \mu_2^{-1} \exp\left(-\frac{x}{\mu_2}\right) \]

\[ x > 0, \mu_1 > 0 (i = 1, 2); \quad p_1 + p_2 = 1. \]

4 (a) Define a mixture of two binomial distributions. Explain the method of moments for estimating the parameters of the distribution. Also deduce the variance–covariance matrix of the estimates.

(b) Define a mixture of two Poisson distributions. Give some applications of the distribution. Describe the method of maximum likelihood in estimating parameters of the distribution.

OR


(b) \[ \mathbf{F} = \left\{ x; x_1 \in R; i = 1, 2 \ldots, n, \sum_{i=1}^{n} x_i^2 \leq w^2 \right\} \]

\[ \tilde{d} = (d_1, d_2, \ldots, d_n) \] is a vector of scalars then show that

\[ I_F = \int_{\mathbf{F}} \cdots \int e^{\tilde{d} \cdot \tilde{x}} d\tilde{x} = \pi^{\frac{n}{2}} \sum_{r=0}^{\infty} \left( \frac{d' \cdot d}{2} \right)^r \frac{(w^2)^n}{r! 2^r \Gamma\left(\frac{n}{2} + r + 1\right)}. \]

5 (a) Write briefly about Frendu’s bivariate exponential distribution.

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(b) Write briefly about Gumble's bivariate exponential distribution.

OR

5 (a) Define truncated bivariate normal distribution. Obtain mean and variance of doubly truncated bivariate normal distribution (when the range of variable \(x\) is truncated from both sides and \(-\infty < y < \infty\)). Hence or otherwise obtain the mean and variance of singly truncated bivariate normal distribution.

\[ h < x < \infty, -\infty < y < \infty \text{ and } -\infty < x < k, -\infty < y < \infty \]

(b) Define truncated trivariate normal distribution. Obtain mean and variance of this distribution.

(c) Define truncated multivariate normal distribution. Obtain mean and variance of this distribution.