Instructions: (1) All questions carry equal marks.
(2) Symbols have their usual meanings.
(3) Calculator can be used.

1. (a) Show that a spinning top initially at rest is released at $\theta_1$ under the action of gravity has a precessional velocity at other angle $\theta_2$ is given by

$$\dot{\phi}_2 = \frac{2mgl}{I_3w_3}$$

(b) Write a short note on limit cycles.

OR

1. (a) (i) What are canonical transformations? Show, by taking suitable generating function, that co-ordinate and momentum are interchangable.

(ii) Show that no two perpendicular components of angular momentum can be taken as canonical momenta simultaneously.

(b) Obtain an exact solution of the non-linear equation

$$\ddot{x} + w_0^2 x - \left(\frac{w_0^2}{\sigma}\right)x^3 = 0$$

Does it give the same result as the method giving an approximate solution.
2 (a) Write short notes on (any \textbf{two}) of the following:

(i) Right ascension and declination co-ordinate system.
(ii) Sun's apparent annual movement on celestial sphere.
(iii) Orbital elements.

(b) An earth satellite has the following elements

\[ T = 3 \text{ hours} \quad e = 0.20 \]

Determine its eccentric anomaly (E) 1 hour after its perigee passage.

OR

2 (a) Derive the following expressions:

(i) \[ r = a(1 - e \cos E) \]

(ii) \[ \tan \left( \frac{f}{2} \right) = \sqrt{\frac{1+e}{1-e}} \tan \left( \frac{E}{2} \right) \]

(b) An earth satellite has the following elements.

\[ T = 12 \text{ hours} \quad e = 0.80 \]

Determine its eccentric anomaly (E) 1 hour after its perigee passage.

3 (a) An infinite straight wire carries the current

\[ I(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ q_0 \delta(t) & \text{for } t > 0 \end{cases} \]

Find the resulting electric and magnetic fields.

(b) Obtain \( \vec{E}, \vec{B} \) for an arbitrary distribution of charges and currents when

\[ V \left( \vec{r}, t \right) = \frac{1}{4\pi \varepsilon_0} \left[ \frac{Q}{r} + \frac{\hat{r} \cdot \vec{p}(t_0)}{r^2} + \frac{\hat{r} \cdot \vec{p}(t_0)}{rc} \right] \]

and

\[ \vec{A} \left( \vec{r}, t \right) = \frac{\mu_0}{4\pi} \frac{\vec{p}(t_0)}{r} \]

H-55019] 2 [Contd...
(c) Discuss radiation damping and obtain

\[ m \left( \vec{V} - \tau \vec{V} \right) = \vec{F}_e \]

where \( \tau = \frac{e^2}{6\pi \varepsilon_0 mc^3} \)

[Hint : \( W = \frac{e^2(\vec{V})^2}{6\pi \varepsilon_0 C^3} \)]

OR

3 (a) If

\[ \frac{dP}{d\Omega} = \frac{1}{4\pi \varepsilon_0} \frac{q^2 a^2}{4\pi c^3} \left[ \left( 1 - \beta \cos \theta \right)^2 - \left( 1 - \beta^2 \right) \sin^2 \theta \cos^2 \phi \right] \]

then find \( P = \frac{1}{4\pi \varepsilon_0} \frac{q^2 a^2}{3 C^3} r^{-4} \)

where \( r = \frac{1}{\sqrt{1 - \beta^2}} \)

(b) find \( \vec{\rho} \) and \( \langle P \rangle \) if

\[ \vec{E} = -\frac{\mu_0 P_0 w^2}{4\pi} \left( \sin \theta \frac{\sin \theta}{r} \right) \cos \theta \left( \frac{t - r}{c} \right) \hat{\theta} \]

and

\[ \vec{B} = -\frac{\mu_0 P_0 w^2}{4\pi c} \left( \sin \theta \frac{\sin \theta}{r} \right) \cos \theta \left( \frac{t - r}{c} \right) \hat{\phi} \]

(c) Discuss dispersion in dilute gases and obtain

\[ n^2 = 1 + \sum_{\alpha} \frac{\left( e^2 / m \varepsilon_0 \right) N f_\alpha}{\left( w_\alpha^2 - w^2 \right) - i l_\alpha w} \]

H-55019] 3 [Contd...
4 (a) (i) Show that the pressure tensor is
\[ P = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} \]
where \( p = nkT \).

(ii) Obtain Liouville equation for \( f^{(N)} \).

(b) Considering the uniform electric field in the region between the plates of a large parallel plate capacitor obtain the transformation rules for electromagnetic fields.

OR

4 (a) (i) Obtain second moment equation.
(ii) Discuss the system of B.B.G.K.Y. equation and obtain.
\[
\frac{\partial f^{(1)}}{\partial t} + \vec{V}_1 \cdot \nabla_{\vec{r}_1} f^{(1)} + \vec{A} \cdot \nabla_{\vec{V}_1} f^{(1)}
+ N \nabla_{\vec{V}_1} \cdot \int a_{12} - f^{(2)} d\vec{r}_2 d\vec{V}_2 = 0
\]

(b) Show that the electric and magnetic fields can be combined into a single entity called field tensor which is an antisymmetric second-rank tensor. Is this tensor gauge invariant?

5 (a) Explain cluster integrals using the method of diagramatic analysis for the evaluation of general virial coefficients for an imperfect gas.

(b) Discuss the phenomenon of gas-liquid condensation.

OR

5 (a) Show that the long range order parameter \( \rho \) goes to zero near curie temperature \( T_C \). The temperature dependence being \( (T_C - T)^{1/2} \).

(b) Explain the term Johnson noise. State and prove Nyquist theorem.