1 - Operations Research

Suggestion:

(1) Write a fuzzy model for the above problem.
(2) Determine the membership functions for the variables.
(3) Formulate the fuzzy optimization problem.
(4) Solve the problem using a suitable fuzzy method.

1) (a) A company produces...

(a) Membership function
(b) Total cost (including)
(c) Membership function of the membership function.
(b) If \( S_F \) does not satisfy the production constraints, the final decision should be made as follows: If \( S_F = \emptyset \), then take the decision. If \( S_F \neq \emptyset \), then solve the problem as above.
(c) The objective function is to be maximized as follows:

Mathematical expression: 

\[ Z = 3x_1 + 2x_2 \]

Subject to:

\[ -2x_1 + x_2 \leq 1 \]

\[ x_1 + x_2 \leq 3 \]

\[ x_1 \leq 2 \]

\[ x_1, x_2 \geq 0 \]

Conclusion...
1. (अ) व्याख्या आयो : 

(1) बिदृत मूल उदेश
(2) प्रारंभिक उदेश

(ब) नींवनी सुरूज आयोजन समस्याने विमंडलेच रूपांत उदेश : 

न्यूनतम करें : 
\[ Z = x_1 - x_2 + x_3 + x_4 + x_5 - x_6 \]

जवः 
\[ -8x_1 - 3x_2 + 12x_3 + x_4 = 3 \]
\[ x_1 + x_4 + 6x_6 = 9 \]
\[ 4x_1 + x_2 - x_4 + 2x_5 = 5 \]

तथा \[ x_i \geq 0, \quad \forall i = 1, 2, \ldots, 6. \]

(5) नींवनी सुरूज आयोजन समस्याने मोटा \( M \) नींवांत उदेश : 

महत्तम करें : 
\[ Z = 6x_1 + 4x_2 \]

जवः 
\[ 2x_1 + 3x_2 \leq 30 \]
\[ 3x_1 + 2x_2 \leq 24 \]
\[ x_1 + x_2 \geq 3 \]

तथा \[ x_1, x_2 \geq 0 \]

2. (अ) सुरूज आयोजन समस्याने तेना समाङज्ञ उदेश स्वरूपांचे किंव तेना क्षमाचे \( 10 \) व्याख्या? आयोजन समस्याने हांड व्याख्यांत करें. वर्गी साखिल करें की हांडने हांड अध मूल समस्या एस. 

(ब) हांडनामा सिद्धीला उपयोग करोयने नींवनी सुरूज आयोजन समस्या उदेश : 

न्यूनतम करें : 
\[ Z = 4x_1 + 3x_2 + 6x_3 \]

जवः 
\[ x_1 + x_3 \geq 2 \]
\[ x_2 + x_3 \geq 5 \]

तथा \[ x_1, x_2, x_3 \geq 0 \]

अध्ययन
2 (a) Cutting plane method

(b) Cutting plane method - Revised formulation

(a) Reformulation

Let \( Z = 4x_1 + 3x_2 \)

where \( x_1 + 2x_2 \leq 4 \)

\( 2x_1 + x_2 \leq 6 \)

type \( x_1, x_2 \geq 0 \), then \( x_1, x_2 \) are non-negative.

3 (a) Generalized formulation

(1) Difficult problems: The generalized formulation involves nonlinear equality constraints.

(2) Difficult problems: The generalized formulation involves nonlinear equality constraints.

(3) Difficult problems: The generalized formulation involves nonlinear equality constraints.

(a) Revised formulation

The final tableau:

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<td>( O_4 )</td>
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Transit

7 9 18 34

Another

(a) This problem involves a large number of equality constraints. The revised formulation provides a more general approach.

5 \times 5 problems - Revised formulation

NE-603] 3 [Contd...
## (a) Analysis of Games:

1. Zero-sum game (zero – sum game)
2. Mixed strategy (mixed strategy)
3. Symmetric game (symmetric game)

4. (a) Complete the following table:

<table>
<thead>
<tr>
<th></th>
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</tr>
</tbody>
</table>

5. (a) Find the solution to the following game:

\[
\begin{pmatrix}
A & B & C \\
A_1 & 0 & 4 \\
A_2 & 6 & -2 \\
A_3 & 3 & -1
\end{pmatrix}
\]

6. (a) For the given game, find the values of x and y that maximize the objective function.

\[
x \cdot \beta + y \cdot \gamma \rightarrow \text{max}
\]

NE-603 [Contd...]
\[
\min_{x, y} f(x, y) \quad \text{subject to} \quad x \leq 0, \quad y \leq 0
\]

\[f(x, y) = \max \min_{x, y} f(x, y)\]

(\text{a})  \text{Give the } 2 \times 4 \text{ matrix to be solved:}

\begin{align*}
\text{B} & \quad \text{B} & \quad \text{B} & \quad \text{B} \\
\text{A} & \quad \begin{bmatrix} A_1 & B_1 & B_2 & B_3 & B_4 \\
        A_2 & 4 & -2 & -3 & 1 \\
        A_3 & 1 & 4 & 5 & 2 \\
        A_4 & 2 & 3 & 4 & 5 \\
\end{bmatrix}
\end{align*}

(\text{b}) \text{Dominance} (\text{Dominance}) \text{ matrix to be solved:}

\begin{align*}
\text{B} & \quad \text{B} & \quad \text{B} & \quad \text{B} \\
\text{A} & \quad \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\
        A_1 & 1 & 4 & 3 & 1 \\
        A_2 & 1 & 5 & 3 & 2 \\
        A_3 & 9 & 1 & 5 & 3 \\
        A_4 & 5 & 4 & 5 & 4 \\
\end{bmatrix}
\end{align*}

(\text{a}) \text{Some problems:}

(1) \text{Convex function}

(2) \text{Lagrangian function}

(3) \text{Quadratic Programming Problem}

(\text{b}) \text{The problems given above are solved using the \textit{Quadratic Programming} method.}

\begin{align*}
\text{Maximize } Z & = 2x_1 + x_2 - x_1^2 \\
\text{Subject to:} & \quad 2x_1 + 3x_2 \leq 6 \\
& \quad 2x_1 + x_2 \leq 4 \\
& \quad x_1, x_2 \geq 0
\end{align*}

NE-603] 5 [Contd...
1 - Operations Research

Instructions: (1) All questions are compulsory.
(2) Figures to the right indicate marks of the corresponding question.
(3) Notations are usual, where not mentioned.
(4) Write the correct number of the question in your answer book as shown in the question paper.

1 (a) Define:
(1) Convex set
(2) Basic feasible solution
(3) Vertex of the convex set.
(b) If \( S_F \) is the set of all feasible solutions of a linear programming problem and if \( S_F \neq \phi \), then prove that \( S_F \) must be a convex set.

(c) Use the graphical method to solve the following L.P. problem:
Maximize : \( Z = 3x_1 + 2x_2 \)
Subject to \(-2x_1 + x_2 \leq 1\)
\(x_1 + x_2 \leq 3\)
\(x_1 \leq 2\)
and \( x_1, x_2 \geq 0 \)

OR

1 (a) Define:
(1) Degenerate basic solution
(2) Optimum solution.

(b) Use the simplex method to solve the following L.P. problem:
Minimize : \( Z = x_1 - x_2 + x_3 + x_4 + x_5 - x_6 \)
Subject to \(-8x_1 - 3x_2 + 12x_3 + x_4 = 3\)
\(x_1 + x_4 + 6x_6 = 9\)
\(4x_1 + x_2 - x_4 + 2x_5 = 5\)
and \( x_i \geq 0 \), \( \forall i = 1, 2, \ldots, 6. \)

(c) Use the Big-M method to solve the following L.P. problem:
Maximize : \( Z = 6x_1 + 4x_2 \)
Subject to \( 2x_1 + 3x_2 \leq 30 \)
\( 3x_1 + 2x_2 \leq 24 \)
\( x_1 + x_2 \geq 3 \)
and \( x_1, x_2 \geq 0 \)
2 (a) When a L.P. problem is said to be in a standard primal form? Define the dual of such a problem. Also prove that the dual of the dual is the primal.

(b) Use the principle of duality to solve the following L.P. problem

Minimize: \( Z = 4x_1 + 3x_2 + 6x_3 \)
Subject to \( x_1 + x_3 \geq 2 \)
\( x_2 + x_3 \geq 5 \)
and \( x_1, x_2, x_3 \geq 0 \)

OR

2 (a) Explain the cutting plane method for solving an integer linear programming problem.

(b) Solve the following integer linear programming problem using the cutting plane method:

Maximize: \( Z = 4x_1 + 3x_2 \)
subject to \( x_1 + 2x_2 \leq 4 \)
\( 2x_1 + x_2 \leq 6 \)
\( x_1, x_2 \geq 0 \) and are integers

3 (a) Attempt any two:

(1) Give a mathematical representation of a transportation problem.

(2) Explain the difference between a transportation problem and an assignment problem.

(3) Describe the method for solving an assignment problem.

(b) Find the optimum solution of the following transportation problem:
(b) Find the minimum cost solution for 5 × 5 assignment problem whose cost coefficients are given as under:

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4 (a) Define:

1. Zero-sum game
2. Mixed strategy
3. Symmetric game.

(b) Prove that the symmetric game has value zero.

(c) Transform the following game into an equivalent L.P. problem (solution is not required):
OR

4 (a) Let the function \( f_{x,y} \) be such that both \( \min_{x,y} \max_{x,y} f_{x,y} \) and \( \max_{x,y} \min_{x,y} f_{x,y} \) exist. If \( (x_0, y_0) \) is a saddle point of \( f_{x,y} \) then prove that

\[
\min_{x,y} \max_{x,y} f_{x,y} = \max_{x,y} \min_{x,y} f_{x,y}
\]

(b) Solve the following 2 × 4 game graphically:

(c) Using principles of dominance solve the following game:

NE-603] 10 [Contd...
5 Attempt any three:

(a) Explain:
(1) Convex function
(2) Lagrangian function
(3) Quadratic Programming Problem.

(b) State the Kuhn–Tucker theorem and derive the Kuhn–Tucker conditions for the following quadratic programming problem:

Maximize: \( Z = 2x_1 + x_2 - x_1^2 \)

Subject to \( 2x_1 + 3x_2 \leq 6 \)
\( 2x_1 + x_2 \leq 4 \)
and \( x_1, x_2 \geq 0 \)

(c) Solve the following nonlinear programming problem graphically:

Maximize: \( Z = 2x_1 + 3x_2 \)

Subject to \( x_1^2 + x_2^2 \leq 20 \)
\( x_1 \cdot x_2 \leq 8 \)
and \( x_1, x_2 \geq 0 \)

(d) Explain:
(1) Carrying (Holding) cost
(2) Ordering cost.

And in usual notations derive

\[ TIC^* = \sqrt{\frac{2D}{C_p \cdot C_h}} \]

(e) A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month's requirement at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying cost is 15% of the average inventory per year. Calculate:
(1) Optimal lot size \( Q^* \)
(2) Minimum yearly total incremental inventory cost \( TIC^* \).
2 - Mechanics

मुद्रण: (1) भव्य ज प्रम्पोऽच उत्तर आपो.
(2) प्रत्येक प्रम्पोऽच गुण २१ छ.
(3) जमीनी बाहुऽच अंक संभवत प्रम्पोऽच गुण दाखऽवे छ.

(1) \( P \) अने \( Q \) भाजनांचे बाणे आंक कल उपर आंकवीर्त तसे \( \theta \) पूऽले कर्य करे छ. तेमनु परिशिचाली भर \( R \), \( P \) साथे \( \alpha \) पूऽले कर्य कर्यु लोऽप तो सापूऽलत करे के

\[
R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad \text{अने} \quad \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}
\]

(अ) अंक समतुल्य एक धार्मिक आलत आले प्रायंतर मातेनु परिशिचाल

(6) \( P \) आपसी आधार नीचे अंक कल समतुल्यांस छ. आधारच आपसी बाणे कोणत्या भरानांमाते \( \theta \) \( \theta \) ती फल कल समतुल्यांमाते रले छ. सापूऽलत करे के जो आधार बाणे कर्य्य रल्यांमाते आले तो फले ते समतुल्यामाते रल्यां.

(2) \( \alpha \) अंक भार रल्यांकी \( 2b \) बाणाईं कल उपर बे छैदे \( W_1 \) अने \( W_2 \) वजनज्ञ सेका बापर्मेवा छ. जेणे भार शेल्टी रल्यां तेमा A बिजल्यांकार्य अंक गोकामे \( b \) मूळवां माल्यां आलोऽपे के, \( W_1 \), वजनज्ञ सेका भागी नीचे शिर्त रल्यां तो सापूऽलत करे के

\[
\mu \alpha^2 = W_2 \epsilon d^2 - a^2
\]

(3) अंक बाणे कोणत्या अंक कल संख्यांतर छ. काटपूऽले कर्य करतांने बे बाणमाते \( W_1 \) अंक वजनज्ञ सेका भाग गलेने छ. गोकामे \( 10 \) फॉल्न लङ्व तो बाकीमा बे बाणोऽच माल रोऽपे.

(4) \( \phi \) \( V = 2 \pi r^4 - x^2 y \) लङ्व तो \( (2, -2, -1) \) प्याल्ये \( \text{Grad} \ V \) अने \( |\text{Grad} V| \) रोऽपे.
2. (अ) पोताला वजन कर्ण मुक्त रूप होते वत्किता समांग देखून घरे स्थान वर्ण नुअं कर्नेत्यिम न्येनु।

अवधारणा

(अ) प्रविष्ट हे संदेशमा दोहरिने हे परवाहक वर्ण स्थः कर्ण अवधारणा

\[ T = T_{0} e^{t} \] छ।

(ब) कोई पक्ष बेना उत्तर आए।

14

(1) सालिन करो के ई लिखितमा अर्थवाचिकाकर केण्टा केन्द्र अन्य द्विध्वज

\[ \text{केन्द्र व्यें} \] अंतर \[ \frac{4\pi}{3\pi} \] छ।

(2) हीसी दीर्घता अन्य भरणगरी जगीत्याची वस्त्रे एक सोल्येन देखण्यामा

आए. ते सोल्या परं एक माल्य वस्त्र छे. ते सोल्या वस्त्रे ते पन्न्यां

टे माल्य देखी सोल्या वस्त्रे ?

(3) सर्वांत विषयाचिता जे लिहितर वस्त्रे एक 30 मीटर वाच वाचे छ।

तेनी महत्तम देखी 3 मीटर छ। वाचर्णु वजन 10 दिलोमां ते

तेनी वस्त्रां उपर एक सर्वपु वर्णरायेच छ। ते वाचर्णा महत्तम तलाव

शोधा।

3. (अ) एक महत्तम गती करता गण्य क्षेत्र अनेक विषयाची अनेक

अधिकांशी घटके भेजया।

अवधारणा

(अ) क्षे संकेतित मदे, गतिशक्तिमां वपारो, अनेक कर्णे कार्य भरणे होय छे

तेम सालिन करो। आ पर्यंत गती संरक्षणमा निभुन तात्त्यो।

(ब) गमे ते बेना जवाब आए।

14

(1) भरक पर सरकनार, खिले विशिष्टां शुरू करीने, 280 मीटर वाचा,

समसंविध साथ \[ \sin^{-1} \frac{5}{14} \] बूले समेवी कान पर, नीचे तरक सरकी

आए छे। जे सरकनार अने भरक वर्णे वर्णांक \[ \frac{1}{\sqrt{19}} \] होय तो कानता

तणिये सरकनाराची उठाम शोधा।
(2) एक निश्चित उद्देश्यवादी दौरान सटिल तिरंगानी दिशायें अने
तेने वर्त दिशायें वेगाना घटको $r^2$ अने $\mu^2$ छ. कलना प्रयोगाना
अर्थय अने अनुपात घटकोने $r$ अने $\theta$ मां अभिव्यक्त करो।
(3) एक पवनां गति करता कलना स्पर्शानी अने अविनमश्रीय प्रयोगाना घटको
सर्पणा मापना छ. जो पवनां स्पर्शकों कोणीय वेग अवय लाग्ने तो
तेने पवनु समीक्षा शोधो।

4 (अ) प्रयत्नित संदर्भांमा $\frac{d^2 u}{du^2} + u = \frac{P}{h^2 u^2}$ समिति करो।

अन्यथा

(अ) वैदिक अध्याय 4 पृष्ठानुसार मान्यता भाष्य पर एक कलने $V$ वेगाधी
केवलां आवे छ. कलना :
(1) वैदिक विश्लेषण
(2) उपभोगनो समय तथा
(3) महत्तम विश्लेषणां सूची भेजनो।

(ब) जगमे तेने ब्रह्मोसा ज्ञान आपोः

(1) एक कलनी हेमिस्फेरिया $r^2 = a^2 \cos^2 \theta$ छ. तेनु वर्ण देखि शुद्ध
छ. समिति करो के भवनो नियम $r^7$ न्यूक अभारां बाद्य छ।
(2) समतल समान्यी $h$ विश्लेषणे आवे पर एक ठेकी उपर एक बंदूक
देखि आवे जो $\theta$ पृष्ठानुसार आवे अने भने प्रश्न
वेग $V$ होय तो तेने महत्तम वैदिक अंतर आयो छ। समिति करो के
\[
cosec^2 \theta = 2 \left( \frac{gh}{V^2} \right) (जवनो अवयो अवगताननो)
\]
(3) Assume a scenario where two objects are involved in a circular motion. The equation for the centripetal force is given by 

$$F = m \cdot \frac{v^2}{r}$$

where $F$ is the centripetal force, $m$ is the mass of the object, $v$ is the velocity, and $r$ is the radius of the circular path.

The centripetal acceleration is given by 

$$a = \frac{v^2}{r}$$

The tangential component of acceleration is given by 

$$\alpha = \frac{v^2}{r}$$

The normal component of acceleration is given by 

$$\beta = \frac{v^2}{r}$$

The total acceleration is given by 

$$\sqrt{\alpha^2 + \beta^2}$$

The centripetal force is given by 

$$F = m \cdot \frac{v^2}{r}$$

The tangential component of force is given by 

$$F_{tan} = m \cdot \alpha$$

The normal component of force is given by 

$$F_{nor} = m \cdot \beta$$

The total force is given by 

$$F = \sqrt{F_{tan}^2 + F_{nor}^2}$$

(4) A ball of mass $m$ is thrown with velocity $v$ at an angle $\theta$ with the horizontal. The initial horizontal component of velocity is $v \cos \theta$ and the initial vertical component of velocity is $v \sin \theta$.

The horizontal component of velocity remains constant, while the vertical component changes due to gravity. The equation for the horizontal component of velocity is 

$$v_x = v \cos \theta$$

The equation for the vertical component of velocity is 

$$v_y = v \sin \theta - gt$$

where $g$ is the acceleration due to gravity and $t$ is the time.

The horizontal distance traveled by the ball is given by 

$$x = v_x \cdot t$$

The vertical distance traveled by the ball is given by 

$$y = v_y \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

The maximum height reached by the ball is given by 

$$y_{max} = \frac{v_y^2}{2g}$$

The maximum range reached by the ball is given by 

$$x_{max} = \frac{v^2 \sin 2\theta}{g}$$

The time of flight is given by 

$$t_{flight} = \frac{v \sin \theta}{g}$$

(5) A projectile is launched with an initial velocity $v$ at an angle $\theta$ with the horizontal. The equation for the horizontal component of velocity is 

$$v_x = v \cos \theta$$

The equation for the vertical component of velocity is 

$$v_y = v \sin \theta - gt$$

The horizontal distance traveled by the projectile is given by 

$$x = v_x \cdot t$$

The vertical distance traveled by the projectile is given by 

$$y = v_y \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

The maximum height reached by the projectile is given by 

$$y_{max} = \frac{v_y^2}{2g}$$

The maximum range reached by the projectile is given by 

$$x_{max} = \frac{v^2 \sin 2\theta}{g}$$

The time of flight is given by 

$$t_{flight} = \frac{v \sin \theta}{g}$$
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2 - Mechanics

Instructions: (1) Attempt any five questions.
(2) Each question carries 21 marks.
(3) Figures to the right indicate full marks of the corresponding question.

1 (a) If two forces $P$ and $Q$ act on a particle, at an inclination $\theta$ to each other and inclination between their resultant $R$ and force $P$ is $\alpha$ then prove that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad \text{and} \quad \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

OR

(a) Prove in the usual notations that for an infinitesimal displacement of a rigid body in a plane

$$\delta w = X \delta a + Y \delta b + N \delta \theta.$$ 

(b) Attempt any two:

(1) A particle is equilibrium under the action of six forces. Three of these forces are reversed, the particle remains in equilibrium. Prove that it will remain in equilibrium if these three forces are removed altogether.

(2) A light rods of length $2b$, terminated by heavy particles of weight $w_1$ and $w_2$ is placed inside a smooth hemisphere bowl of radius $a$ which is fixed with its rim horizontal. If the particle of $w_1$ rests just below the rim of bowl then prove that

$$w_1a^2 = w_2b^2 - a^2.$$ 

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(3) A particle is in equilibrium under three forces. Two of the forces act at right angles to one another, one being three times than the other. The third force has a 10 pound weight. Find the magnitude of the other two.

(4) Find \( \nabla V \) and \( |\nabla V| \) at a point 
\[ (2, -2, -1) \] if \( V = 2xz^4 - x^2y \).

2 (a) Obtain the Cartesian equation of a curve formed by a uniform cable hanging freely under its own weight.

OR

(a) In usual notations show that the tension is \( T = T_0 e^{2\theta} \) if a light cable rests in contact with rough curve.

(b) Attempt any two:

1. Prove that the distance between centre and the mass centre of a semicircular plat of radius \( a \) is
\[ \frac{4a}{3\pi} \]

2. A ladder is supported on a smooth floor and leans against a rough wall. How far, a man can climb up the ladder, without slipping taking place?

3. A suspension cable with supports the same level has a span of 30 meter and a maximum dip of 3 meter. The cable is loaded with a uniformly distributed load of 10 kilogram throughout its length. Find the maximum tension in the cable.

3 (a) Find the tangential and normal components of velocity and acceleration of the particle moving in a plane.

OR
(a) For a system of particles, prove that the increase in kinetic energy is equal to the work done by force. From this deduce the principle of conservation of energy.

(b) Attempt any two:

1. A skier, starting from rest, descends a slope 280 meter long and inclined at an angle of $\sin^{-1}\frac{5}{14}$ to the horizontal. If the coefficient of friction between the skier and the snow is $\frac{1}{\sqrt{19}}$, find the speed of the skier at the bottom of the slope.

2. The components of velocity of a particle along the direction of a radius vector drawn from a fixed origin and perpendicular to it are respectively $\lambda r^2$ and $\mu \theta^2$. Express radial and transverse components of its acceleration in terms of $r$ and $\theta$.

3. The tangential and normal components of acceleration of a particle moving along a curve are equal in magnitude. If the angular velocity of the tangent to the curve is constant then find the equation of the curve.

4. In usual notations, prove: $\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}$.

OR

(a) A particle is projected with velocity $V$, making an angle $\alpha$ with the horizontal. Find the expressions for

1. the horizontal range

2. the time of flight and

3. the maximum height attained by the particle.
(b) Attempt any two:

1. A particle describes the central orbit \( r^2 = a^2 \cos 2\theta \), the centre of force being the pole. Show that the law of force varies inversely as \( r^7 \).

2. A gun is mounted on a will of height \( h \) above a level plain. Show that the greatest horizontal range for a given muzzle velocity \( V \) is obtained by firing at an angle of elevation \( \theta \) such that

\[
\csc^2 \theta = 2 \left( \frac{gh}{V^2} \right) \]

(Neglecting resistance of air)

3. A particle executes simple harmonic motion such that in two of its positions the velocities are \( u \) and \( v \) and corresponding accelerations are \( \alpha \) and \( \beta \). Show that the distance between the positions is

\[
\frac{v^2 - u^2}{\alpha + \beta} \]

Also show that the amplitude of the motion is

\[
\frac{\left[ \varepsilon^2 - u^2 \right] \Phi^2 \left( \frac{v^2 - \beta^2 u^2}{\varepsilon^2 - \alpha^2} \right)^{1/2}}{\beta^2 - \alpha^2} \]

5. (a) A solid cylinder of radius \( r \) and mass \( m \) is rolling down a plane inclined at an angle \( \alpha \), prove that its acceleration is \( \frac{2}{3} g \sin \alpha \).

OR

(a) Two spheres of masses \( m_1 \) and \( m_2 \), moving in a straight line with velocities \( u_1 \) and \( u_2 \) collide with each other. If \( e \) is the coefficient of restitution, obtain their velocities after collision.
(b) Attempt any two:

1. Find the moment of inertia of a solid sphere of mass \( m \) and radius \( a \) about its diameter.

2. A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution is \( e \), show that their velocities after impact are in the ratio \( \frac{1-e}{1+e} \).

3. A rigid body of mass \( M \) is fastened to one end of a light thread, the thread is coiled round a windless in the form of circular cylinder of radius \( a \) which is left free to rotate about its axis. Prove that the rigid body descends with acceleration

\[
\frac{g}{1 + \frac{I}{Ma^2}}
\]

where \( I \) is the moment of inertia of the cylinder about its axis.

---

3 - Number Theory

सूचना : (1) प्रशंसित संख्यात अनुसरण.
(2) जमशेदी बाणी अंक अनुसूच न पेट्र-प्रश्न भव दशावे छे.

1. (अ) \( p, p+2 \) अने \( p+4 \) अंग्ष्ठ अविभाज्य संख्यांसह होप तेवा तमाम \( p \) शोधो.
   (ब) अने शून्य न होप तेवा पूर्णाक \( a, b \) आपेक्षा छे. साधित करो ए
   अंवा पूर्णाक \( x \) अने \( y \) अस्तित्व पहाउँ छे \( \gcd(a, b) = ax + by \).
   (ग) अंकप्रशिति मूलानुमूं प्रमेय जपो अने साधित करो.

अध्याय
1 (अ) यदि $n > 4$ विभाज्य संख्या लाख तो सामिल करो $k = (n-1)$ ने $n$ वे भिन्न करे।
(ब) $4n+3$ स्वप्नानी अविभाज्य संख्याओं अन्तर छ तेम भलावो।
(त) प्रमेय-कोज्योन संमीकरण $24X + 138Y = 18$ ना तभाय पुलिंड उड़े दो में।

2 (अ) $1! + 2! + 3! + \ldots + 99! + 100!$ ने 12 ये भागता भजनी लेश शोपी।
(ब) सामिल करे के सुरेश समांथाता संमीकरण $ax \equiv b \pmod{n}$ ने उड़े लाख तो अने तो ज $d \mid b$, तथापि $d = \gcd(a, n)$.
(त) $7 \times a$ लाख तो सामिल करो $k = a^3 + 1$ अवश्य $a^3 - 1$ ने 7 ये निषेध भजनी लाखाय।

अवश्य

2 (अ) यदि $p$ अने $q$ विभाज्य अविभाज्य संख्याओं अने के छेड़ी

$$a^p \equiv a \pmod{q} \text{ अने } a^q \equiv a \pmod{p} \text{ शास्त्री। सामिल करे के}

a^{pq} \equiv a \pmod{pq}.$$

(ब) यदि $p$ अविभाज्य संख्या होय तो भलावो $k : (p-1)! \equiv -1 \pmod{p}$.
(त) सुरेश समांथाता संमीकरण $9x \equiv 21 \pmod{30}$ ने उड़े।

3 (अ) $\Omega(n)$ अने $\sigma(n)$ वापसपायित करे。

यदि $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$ अने $n > 1$ विभाज्य धात अवश्य करो लाख तो सामिल करो $k$ :

$$\Omega(n) = \prod_{i=1}^{r} \left( p_i^{k_i+1} - 1 \right) / \left( p_i - 1 \right)$$

तार्क के $\Omega$ अने $\sigma$ मैटेड्यून्कटिव विशदाय।

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(a) \( n>2 \) मात्र अंतर्गत के \( \phi(n) \) अंक एकी संख्या छ.  
(2) \( k \) के लिए \( \forall \) पूर्णांक \( n \) मात्र अंतर्गत के 
\[ \mu(n) \cdot \mu(n+1) \cdot \mu(n+2) \cdot \mu(n+3) = 0. \]

अवधि

(3) \( \phi \) के \( n \) अंक \( r \) पूर्णांक \( n \) हो, \( 1 \leq r < n \) तो साधित करें के

\[ \text{दिया गया सिद्धान्त: } \prod_{r=1}^{n} \frac{n!}{(n-r)!r!} \text{ गदा अंक पूर्णांक हैं.} \]

(4) \( \text{मोऽबीयस } \phi \text{-विषय व्याख्या करें.} \]

(5) \( \text{साधित करें के } k \text{ का महत्त्वपूर्ण विषय है.} \]

(6) \( \text{पूर्णांक } n \text{ मात्र अंतर्गत के } \frac{\sqrt{n}}{2} \leq \phi(n) \leq n. \]

(7) \( \text{अलग-अलग } a \text{ तली कस्ता } k \text{ मोऽबीयस } n \text{ केंद्र अंक } r>0 \text{ होने तो साधित करें के } a^r \text{ तली कस्ता } \frac{k}{\gcd(r,k)} \text{ मोऽबीयस } n \text{ छ.} \]

(8) \( \text{साधित करें के } k \text{ अंक } 19 \text{ मूलबृहत बीज छे ना } 13 \text{ अनुमो.} \]

अवधि

(9) \( \text{मूलबृहत संख्या } M_{13} \text{ अदिविभाज्य संख्या के तेम अंतर्गत.} \]

(10) \( \text{अंक } 2^k - 1 (k>1) \text{ अदिविभाज्य संख्या होने तो साधित करें के } n=2^k-1 (2^k-1) \text{ अदिविभाज्य छ.} \]

(11) \( k \geq 3 \text{ होने तो साधित करें के } 2^k \text{ अंक मूलबृहत बीज नहीं.} \]

(12) \( \text{पूरा करें } p \text{ अंक अदिविभाज्य संख्या छे अंक } \gcd(a,p)=1 \text{.} \]

[वर्तमान होने तो \( a \) अंक \( p \) से वर्तमान अवशेष छे अने तो \( a^{(p-1)/2} \equiv 1 \text{ (mod } p) \).]
3 - Number Theory

Instructions: (1) Follow the usual notations.
(2) Figures to the right indicate the marks of the corresponding sub-question.

1 (a) Find all $p$ for which $p$, $p+2$ and $p+4$ all are primes. 5
(b) Given integers $a$ and $b$ not both of which are zero. 8
Prove that there exist integers $x$ and $y$ such that $\text{gcd}(a, b) = ax + by$. 

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(c) State and prove the fundamental theorem of arithmetic.

OR

1  (a) Prove that if \( n > 4 \) is composite then \( n \) divides \((n-1)!\).  
(b) Prove that there are infinite number of primes of the form \(4n+3\).  
(c) Determine all solution in the integers of the Diophantine equation \(24X + 138Y = 18\).

2  (a) Find the remainder when \(1! + 2! + 3! + \ldots + 99! + 100! \) is divided by 12.  
(b) Show that the linear congruence \(ax \equiv b \pmod{n}\) has a solution iff \(d \mid b\), where \(d = \gcd(a, n)\).  
If \(d \mid b\), then show that it has \(d\) mutually incongruent solution modulo \(n\).  
(c) If \(7 \times a\), prove that \(a^3 + 1\) or \(a^3 - 1\) is divisible by 7.

OR

2  (a) If \(p\) and \(q\) are distinct primes such that \(a^p \equiv a \pmod{q}\) and \(a^q \equiv a \pmod{p}\). Prove that \(a^{pq} \equiv a \pmod{pq}\).  
(b) If \(p\) is a prime then prove that \((p-1)! \equiv -1 \pmod{p}\).  
(c) Solve the linear congruence \(9x \equiv 21 \pmod{30}\).  

3  (a) Define \(\mathcal{I}(n), \sigma(n)\).  
If \(n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}\) is the prime power factorization of \(n > 1\). Then prove that:
\[
\mathcal{I}(n) = [\beta_1 + 1][\beta_2 + 1][\beta_3 + 1] \ldots [\beta_r + 1]
\]
and
\[
\sigma(n) = \prod_{i=1}^{r} \frac{p_i^{k_i+1} - 1}{p_i - 1}
\]
Deduce that \(\mathcal{I}\) and \(\sigma\) are multiplicative functions.
(b) For \( n > 2 \) prove that \( \phi(n) \) is an even number.

(c) For each positive integer \( n \), show that
\[
\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0.
\]

**OR**

3  (a) If \( n \) and \( r \) are positive integers with \( 1 \leq r < n \), then prove that the binomial coefficient
\[
\binom{n!}{(n-r)!r!}
\]
is also an integer.

(b) Define the Mobius \( \mu \)-function. Show that it is multiplicative function.

(c) For a positive integer \( n \), show that \( \frac{\sqrt{n}}{2} \leq \phi(n) \leq n. \)

4  (a) If the integer \( a \) has order \( k \) modulo \( n \) and \( r > 0 \).
Prove that \( a^r \) has order \( \frac{k}{\gcd(r, k)} \) modulo \( n \).

(b) Show that 2 is a primitive root of 19, but not of 17.

(c) Show that the Fermat number \( F_5 \) is divisible by 641.

**OR**

4  (a) Prove that the Mersenne number \( M_{13} \) is a prime.

(b) If \( 2^k - 1 \) is a prime \( (k > 1) \), then prove that
\[
n = 2^{k-1}(2^k - 1)
\]
is a perfect number.

(c) For \( k \geq 3 \) prove that the integer \( 2^k \) has no primitive roots.

5  (a) Let \( p \) be an odd prime and \( \gcd(a, p) = 1. \)
Prove that \( a \) is a quadratic residue of \( p \) iff
\[
a^{(p-1)/2} \equiv 1 \pmod{p}.
\]
(b) If $p$ is an odd prime then prove that the Legendre symbol

\[
\begin{align*}
\left\langle \frac{-\Phi}{\eta} \right\rangle &= 1 & & \text{if } p \equiv 1 \pmod{4} \\
\left\langle \frac{-\Phi}{\eta} \right\rangle &= -1 & & \text{if } p \equiv 3 \pmod{4}
\end{align*}
\]

(c) Prove that if $x, y, z$ is a primitive Pythagorean triples in which the difference $z - y = 2$, then $x = 2t$, $y = t^2 - 1$, $z = t^2 + 1$, for some $t > 1$.

**OR**

5 (a) If $x, y, z$ is a primitive Pythagorean triple, prove that $x + y$ and $x - y$ are congruent modulo 8 to either 1 or 7.

(b) Let $p$ be an odd prime and $a$ and $b$ be integers which are relatively prime to $p$. Prove that if $a \equiv b \pmod{p}$ then

(c) Show that 3 is a quadratic residue of 23, but a non-residue of 31.

**4 - Pascal Programming**

**Instructions**: (1) All questions are **compulsory**.

(2) Figures on the right indicate the marks of the question.

(3) Write correct number of the question in your answer book as shown in the question paper.
(a) Attempt any three:

1. If a:=−17 div 4; b:=−17 mod 4 c:=17 mod(−4) then what are the values of abs(trunc(a)), round(abs(c−b)) and round(abs(trunc(a+2b+c)))?

2. Determine which of the following are valid numbers with proper reason. If a number is valid, specify whether it is integer or real:

0.521, +93e12, −5.83e−67, 1., 42−55, 12e2, 131073, 1.31572e5.

(b) For a given positive integer n, write a program to print the following digit pyramid of order n as follows. Print the output in the following manner: (assume that n<=9).

1
1 2 1
1 2 3 2 1
1 2 3 4 3 2 1
1 2 3 4 5 4 3 2 1

(c) A given positive integer n is said to be perfect if the sum of all the positive divisors of n is equal to 2n. e.g. 6 is a perfect number 2(6)=1+2+3+6. Write a program that given positive integer is (i) prime (ii) perfect or not.

(d) Write a program (i) to exchange the values of the two variables using only two variables. Write a program (ii) to accept three numbers a, b and c (iii) to change the values of a, b and c (iv) to print the new values of a, b and c.
(e) Write a program to evaluate the following series correct to four decimal digits:

\[(1) \quad -\frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\]

\[(2) \quad 1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \ldots \quad \text{where} \quad |x| > 1.\]

2 Attempt any three:

(a) Write an algorithm and program to find an integer in the range 50 to 100 having the maximum number of divisors.

(b) Write a function subprogram to evaluate the factorial of a given non-negative integer \( n \). Using this function subprogram write a program to evaluate the binomial coefficient \( \binom{n}{r} \) for given non-negative integers \( n \) and \( r \) with \( 0 \leq r \leq n \).

(c) What will be the output of the following program segment? (Select from the option with proper explanation.)

```
Procedure swap(name x, y: integer);
    var z: integer;
    being
    z := x; x := y; y := z;
end;
```
with respect to the above procedure of some main program what is the effect of the following ? a[1]=2, a[2]=5 and i=1 :

(1) What would be the effect of swap(a[i], i) ?

(2) What would be the effect of swap(a[i], i) ?

(3) What would be the effect of swap(a[i], i), if x, y in swap are variable parameters ?

(4) What would be the effect of swap(a[i], i), if x, y in swap are value parameters ?
(5) What would be the effect of swap(a[i], i), if x, y in swap are value-result parameters?

(i) \( a[1] = 1, a[2] = 5 \) and \( i = 1 \).

(ii) \( a[1] = 1, a[2] = 5 \) and \( i = 2 \).

(iii) \( a[1] = 2, a[2] = 1 \) and \( i = 2 \).

(iv) \( a[1] = 2, a[2] = 1 \) and \( i = 1 \).

(d) Explain the control structures WHILE-DO, REPEAT-UNTIL, FOR-TO, FOR-DOWNTO. Use the four structures to find the summation of the first \( n \) positive integers, where \( n \) is a given positive integer.

(e) Answer as true or false:

(i) Pascal does not allow the use of lower case letters.

(ii) The ordinal numbers of the Boolean constants true and false are identical.

(iii) Extremely small numbers cannot be written in the exponent form.

(iv) Blanks are allowed in an identifier.

(v) Pascal does not allow the use of any other data type except scalar.

(vi) Constant declaration may follow the variable declaration.

3 Attempt any three:

(a) In the context of:

\[
\text{var } b: \text{array}[1..n,1..n] \text{ of integer}; i, j: \text{integer;}
\]
The following code segment;

for i := 1 to n do
for j := 1 to n do
b[i, j] := (i div j) * (j div i);

Is equivalent to:

(x) for i := 1 to n do
    for j := 1 to n do
        b[i, j] := 1;

(y) for i := 1 to n do
    for j := 1 to n do
        b[i, j] := 0;
    for i := 1 to n do
    for j := 1 to i do
        b[i, j] := 1;

(z) for i := 1 to n do
    for j := 1 to n do
        b[i, j] := 0;
    for i := 1 to n do
    b[i, i] := 1;

(w) for i := 1 to n do
    for j := 1 to n do
        b[i, j] := 0;
(b) Write an algorithm and a program to read array A and B of integers and create a new array C such that element of A and B are alternatively found in C.

(c) Given a randomly ordered set of n numbers sort them into descending order using an insertion method.

(d) Write an algorithm or program to remove all duplicates from an ordered array and contract the array accordingly.

(e) Design an algorithm to find the number of times the maximum occurs in an array of n elements. Only one pass through the array should be made.

4 Attempt any two :

(a) Explain: Static variables, dynamic variables, advantages and disadvantages of the pointer data type variables with proper examples.

(b) Design and implement a recursive algorithm to solve the Towers of Hanoi problem for one or more disks.

(c) Write an algorithm and program using recursive approach to find the GCD for two positive integers and to find the factorial of a positive integer.

(d) Design and implement a recursive quicksort algorithm to sort an array of integers.

(e) Design and implement procedures that maintain a queue that can be subjected to insertion and deletions.