

GUJARAT UNIVERSITY
Syllabus of S.Y.B.Sc. / S.Y.B.A. (Effective from June 2004)
Mathematics Paper-III
Advanced Calculus

Unit-I Indeterminate forms, L'Hospital's rules, Increasing and decreasing functions, Concave upwards and concave downwards functions, Points of inflexion, Asymptotes, Curve tracing (Catenary, Cycloid, Folium, Astroids, Limacon, Cissoid, Cardioid)

Unit-II Real functions of several variables, Their limit and continuity, (Repeated limits and limits in \mathbb{R}^2 to be explained), Partial derivatives of functions of n variables (For special case $n=2$ notation D_{12} and D_{21} to be explained), Differentiability, Chain rule, Partial derivatives of higher order, Condition for commutative property of variables in higher order partial derivatives, Derivatives of implicit functions.

Unit-III (a) Euler's theorems on partial derivatives of homogenous functions, Extrema of functions of several variables, Lagrange's method of undetermined multipliers, Taylor's and Maclaurin's expansions for functions of several variables (Proof for case of two variables only)

(b) Curves, Surfaces, First order partial differential equation, Classification of integrals, Linear equations of first order.

Unit IV Differentiation along a curve, Applications to geometry

- (i) Curvature in Cartesian and polar co-ordinates,
- (ii) Singular points for plane curves especially points of inflexion and double points.
- (iii) Tangent line and normal plane to curves .
- (iv) Tangent plane and normal line to surfaces
- (v) Gradient

Unit V (i) Double integral , Repeated integral, integral on a non-rectangular region, Jacobian, (only introduction and use for transformation from Cartesian to polar, spherical and cylindrical co-ordinates and vice-versa.

(ii) Divergence and curl of a vector, Line integral , Surface integral, Green's theorem and Stoke's theorem

The course is roughly covered by

- (i) David Widder: Advanced Calculus (Prentice hall ,New Delhi)
- (ii) T.M. Apostol: Advanced Calculus Volume-II (Blaisdoll)
- (iii) Shanti Narayan: Differential Calculus (S.Chand)
- (iv) Shanti Narayan: Integral Calculus (S.Chand)
- (v) T. Amarnath: Partial Differential Equation (Narosa) (Articles 1 to 4)
- (vi) James Stewart: Calculus Brooks/Cole publishing company
- (vii) S.T. Tan: Applied Calculus Brooks/Cole publishing company

The course of advanced calculus is roughly covered by Chapters 1,2,3,4,6,7 and 8 of book by David Widder.

Paper IV

Linear Algebra

Unit-I Vector space, Definition and examples, Vector Subspaces, Linear dependence and independence, Span of a set, Basis and dimension of a vector space, Line, Affine spaces, Quotient spaces.

Unit II Linear transformation, Representation of linear transformation by a matrix, Kernel and image of a linear transformation, linear isomorphism, Geometric ideas and rank, Identity, Stretch along axes, Reflection with respect to axes, Rotation, Shear, Projection, Their Combinations

Unit III Inner product spaces, the Euclidean plane and the dot product, general inner product spaces, orthogonality, Geometrical application, orthogonal projection onto a line, orthonormal basis, orthogonal complements and projections, linear functionals and hyper-planes, orthogonal transformations, associated co-ordinates, reflections, orthogonal map of the plane.

Unit IV Determinants and its properties, Value of a determinant, Basic results-Laplace expansion, Cramer’s rule, Application to geometry, orientation and vector product.

Unit V Rotation of axes of conics, Eigenvalues and eigenvectors, Cayley –Hamilton theorem, Diagonalization of symmetric matrices, conics and quadrics, classification of Quadrics , computational examples.

The course is covered by

Linear Algebra- A Geometric Approach

S.Kumaresan (PrenticeHall, New Delhi)2003

Reference: (i) Finite Dimensional Vector spaces

P.Halmos

(ii) Matrix and Linear algebra

K.B.Dutta (Prentice Hall, New Delhi)

(iii) Linear Algebra –A problem book

P.R.Halmos(Cambridge university Press)

(iv) Linear Algebra

G.Paria (New central book agency-Calcutta)

(v) Linear algebra and applications

Gilbert Strang Thomson Brooks/ cole

Paper VA

Numerical Analysis and Boolean Algebra

Unit I Error in calculation and calculus of finite differences, interpolation

Significant error, Relative error, Estimation of error, Application of error formula, Forward differences, Backward differences, Shift operator, Polynomial in factorial notation, error in interpolation.

Unit II Interpolation and inverse interpolation: Central difference, Gauss’s forward and backward formula, Stirling’s interpolation formula, Bessel’s and Everett’s formulae, Lagrange’s

formula, Divided difference, Newton's divided difference formula, inverse interpolation, its application.

Unit III Numerical differentiation and integration: Estimation of error in differentiation formula based on Newton's forward and backward formulae, and Stirling's formula, differentiation formulae for un-equispaced arguments, General quadrature formula, Trapezoidal rule, Simpson's rule, Weddel's rule, quadrature formula based on Lagrange's formula, Newton-Cotes formula, Numerical integration formula based on central difference formulae, Euler – Maclaurin sum formula.

Unit IV Algebraic and transcendental equations, Numerical solution of differential equations of first order: Graphical method, method of bisection, method of iteration, Newton-Raphson formula, Newton's iterative formula, method of false position, Euler's method, Euler's modified method, Picard's method, Taylor's series method, Runge-Kutta method, Milne's method.

Unit V Relations, Posets, Hasse diagrams, Lattices as posets and algebraic system, complemented and distributive lattices, Homomorphism and isomorphism of lattices, Stone's representation theorem for Boolean algebra, Boolean expression, binary valuation, Boolean function.

The course of numerical analysis is covered by Chapters 1,2,3,4,5,6,8 of Numerical analysis and computational procedures by S.A.Moolah, New central book agency (p) ltd, 8/1 Chintamani Das Lane, Calcutta 700009.

Discrete mathematical structures with application to computer science

J. P . Tremblay and R. Manohar (Mcgraw Hill) covers Boolean Algebra. (Part of Chapter 4)

Refernce Books:

- (i) Elementary Numerical analysis S.S. Sastry (Prentice Hall, New Delhi)
- (ii) Numerical mathematical analysis Scarborough 6th edition, (Oxford & IBH)
- (iii) Numerical analysis S.Kunz Mcgraw Hill Book Newyork
- (iv) Numeric Analysis Richard Burden and J. Douglas Thomson

Paper VB

Mathematics Practicals

Unit I: (a) Tracing of curves and graphical solutions:

- (1) All trigonometric functions and their inverse functions
- (2) Quadratic, cubic, exponential and logarithmic functions.

(3) Graphs of $xy=1, xy=-1, y=ax^2+bx+c, y = \frac{x+a}{x-a}, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

- (4) Parametric equations: Astroid, Hyperbola, Ellipse, Parabola, Cycloid
- (5) Polar forms: Cardioid, circle, conic, Spiral, limaçon, Lemniscate of Bernoulli.
- (b) Hasse Diagrams of (S_{30}, D) , (S_{210}, D) , (S_{12}, D) etc.

Unit II: Interpolation and inverse interpolation:

- (1) Interpolation using Newton's forward and backward interpolation formula.
- (2) Interpolation using Gauss's, Stirling's, Bessel's and Everett's interpolation formula.
- (3) Interpolation using Lagrange's equal length and unequal length interpolation formula.
- (4) Interpolation using Newton's divided difference formula.
- (5) Inverse interpolation.

Unit III: Numerical Differentiation and Integration and solution of Differential Equations.

- (1) Numerical differentiation for equi-spaced and unequi-spaced arguments.
- (2) Quadrature formula.
- (3) Picard's and Taylor's method.
- (4) Euler's, Modified Euler's and Runge-Kutta's method.
- (5) Bisection method, method of false position, fixed point iteration, Newton-Raphson method, Horner's method.

Unit IV: System of linear Algebraic equations

- (1) Elimination method
- (2) Inversion of matrix by (a) Row-reduced echelon form (b) Gauss-Seidel iteration method.
- (3) Eigen-values and eigen-vectors in \mathbb{R}^3 .
- (4) Minimum polynomial of 3×3 matrix.
- (5) Canonical and quadratic forms.
- (6) Least square method.

The course of practicals is covered by part II computational procedures by Moolah S.A. Chapters 1,2,3,4,5,6,7,8,10.

There will be four periods a week for paper III and IV and three periods a week for paper VA. The practical of two periods a week for a batch of 20 students for paper VB.

Optional Paper : PaperVA

(Computer oriented Numerical Analysis)

Instructions:(1) The programming language "C" should be used only for syntax and semantics. Detailed non-mathematical examples which divert the attention should be discarded from the discussion.

(2) The syllabus of “C” programming language is covered by

Programming in ANSI C by BALAGURUSWAMY, Tata McGraw-Hill, 2nd edition, Ch. 1-7,9.

(Omit case studies, multidimensional arrays)

(3) For practicals, algorithms are covered by “ Numerical methods for mathematics, science and engineering” by John H Mathews (2nd edition) Prentice Hall of India.

Algorithms- 2.1-2.3, 2.5, 2.6, 3.1, 3.4, 3.5, 4.2-4.5, 6.1, 6.3, 7.1-7.3, 9.1-9.4, 9.7.

(4) Topics of numerical analysis are covered by “Numerical analysis and computational procedure”

By Dr. S.A. Mollah, Books and allied (P) Ltd. Chapters 1-6 and 8. (omit 6.13, 6.14)

Unit I: Introduction, Importance of C, Basic Structure of C programs, Programming style, Executing a C program, Character set, C tokens, Keywords and Identifiers, Constants, Variables, Data types, Declaration of variables, Assigning values to variables, Defining symbolic constants, Arithmetic of operators, Relational, logical assignments, increment, decrement, Conditional Bitwise and special operators, Arithmetic expressions, Evaluation of expressions, Precedence of arithmetic operators, Some computational problems, Type conversions in expressions, operator precedence and associativity, Some mathematical functions, Reading and writing a character, Formatted input and output, Decision making and branching: The IF statement, Simple IF statement, the IF ELSE statement, Nesting of IF ...ELSE statement, The ELSE IF ladder, the switch statements, the ?: operator, The GOTO statement, Decision making and Branching : the WHILE, DO, and FOR statements, jumps in loops, Arrays, One and two dimensional arrays, initializing Two-dimensional arrays.

Unit II : User defined functions, Need for user-defined functions, A multifunction program The form of C functions, Calling a function, Category of functions, No argument and no return values, Argument but no return values, Argument with return values, Handling of non-integer functions, Nesting of functions Recursion, Functions with arrays, The scope and lifetime of a variables in functions, ANSI C functions.

Introduction to the free mathematical software such as “GNU PLOT” to draw the graphs of standard mathematical functions,

Errors: Rounding off, significant, absolute, relative, relative percentage, relation between the relative error and the number of correct, significant figures, (without proof) Estimation of errors,

Finite differences: Forward Δ and backward ∇ , Fundamental theorem of difference calculus,

Shift operator E, its relation with Δ D operator and its relation with ∇ . Factorial notation,

Polynomials in factorial notation.

Unit III: Interpolation and inverse interpolation with equal and unequal intervals, error in interpolation, Newton's forward and backward interpolation formula, (Estimation of error), Central difference interpolation formula: Gauss's forward and backward formula, Stirling's interpolation formula, Bessel's and Everett's formulae, Lagrange's interpolation formula, Divided difference, Newton's divided difference formula, inverse interpolation, its applications.

Unit IV: Numerical differentiation and integration: Estimation of error in differentiation formula based on Newton's forward and backward formulae, and Stirling's formula, differentiation formulae for un-equispaced arguments, Gauss Legendre quadrature formula, Trapezoidal rule, Simpson's rule, Weddel's rule, quadrature formula based on Lagrange's formula, Degree of precision in quadrature formula, Newton-Cotes formula, Numerical integration formula based on central difference formulae, Euler –Maclaurin sum formula.

Unit V Algebraic and transcendental equations, Numerical solution of ordinary differential equations of first order : Graphical methods, method of bisection, method of iteration, Newton-Raphson formula, Newton's iterative formula, method of false position, Euler's method, Euler's modified method for o.d.e., Picard's method, Taylor's series method, Runge-Kutta method, Milne's method.

Optional Paper : VB

Practicals for computer oriented numerical analysis.

Unit I: The graphs (1)sin(x),cos(x),tan(x),cot(x),cosec(x),sec(x)

(2) arcsin(x), arccos(x), arctan(x), arccot(x), arccosec(x), arcsec(x)

(3) e^x, a^x (4)log(x), In(x) (5) Straight lines ,circle. (6)Parabola, ellipse, hyperbola. (7)cycloids, cardioids,astroids. (to be drawn using Gnuplot software.)

Unit II (1)Fixed point iteration. Solve $x=g(x)$ (2) Bisection method to solve $f(x)=0$.

(3)Method of false position to solve $f(x)=0$.(4)Newton-Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

(5)Secant method to solve $f(x)=0$. (6) Back substitution to solve upper triangular system.

(7) Jacobi iteration to solve system of linear equation.(8) Gauss-Seidal iteration to solve system of linear equation.

Unit III (1) To find value of a polynomial P(x) using polynomial calculus(synthetic division)

(2)To find value of derivative of a polynomial P(x) using polynomial calculus(synthetic division)

(3) To find value of integral of a polynomial P(x) using polynomial calculus(synthetic division)

(4)Lagrange approximation (5)Nested multiplication with multiple centre (6)Newton's divided difference interpolation formula

Unit IV : (1) Differentiation using limits (2) Differentiation based on n points (3) Numerical integration using composite trapezoidal rule. (4) Numerical integration using composite Simpson's $1/3$ and $3/8$ rule. (5) Numerical integration using recursive trapezoidal rule. (6) Numerical integration using Weddle's rule. (for six intervals)

Unit V: (1) Euler's method (2) Modified Euler's method (3) Taylor's method of order 4. (4) Runge-Kutta method of order 4 (5) Runge-Kutta method of order 2. (6) Milne-Simpson's predictor corrector method.